

Spin-only approach to quantum magnetism in the ordered stripe phase.

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It has been argued that the spin-dynamics in cuprate superconductors is governed by the proximity to a zero-temperature critical point. This critical point would be related to a transition from the superconducting phase to an ordered stripe state. Using a coupled spin-ladder model, we investigate to what extent the strong quantum spin-fluctuations in the ordered stripe state can be attributed to the spin sector. For the bond-centered stripes, we find that our spin-only model can account for the observed spin-fluctuations. For site-ordered stripes, coupling of the spin and charge sector is needed.

Recent experiments on magnetism in the underdoped cuprates indicate that the spin-sector in these materials is close to a zero-temperature critical point [1,2]. It was suggested by Aeppli *et. al.* that this critical point is due to a transition from the spin-disordered superconducting phase, to a spin-ordered stripe phase [2]. To reach this, an extra parameter has to be tuned, apart from hole-doping. This can be for instance LTT deformation, which stabilizes stripe-ordering, or Zn-doping/magnetic field, which suppresses superconductivity.

The transition seems to be in the universality class of the Quantum Non-Linear Sigma model (QNLS) [1–3]. What the properties of the QNLS are and how they fit the observed behavior will be discussed below. First, we should point out that it is actually not self-evident that this model should capture the long-wavelength magnetic behavior in the superconducting phase. The QNLS is a model of transversal spin fluctuations that has been successfully applied to the antiferromagnetic phase at zero doping [8]. Since the microscopy of the superconducting phase is very different from that of the zero-doping antiferromagnet, it would be remarkable if both systems showed the same magnetic behavior at long distances.

In this paper, we will focus on the stripe phase, at the spin-ordered side of the transition. This phase provides a perspective on how to arrive at the critical behavior, starting from the microscopy. Consider the following cartoon of a charge ordered stripe state. It consists of a regular grid of lines of immobile holes, which form anti-phase domain walls in a two-dimensional antiferromagnet. This picture naturally leads to a spin-only description, since charge and spin are spatially segregated. As Castro Neto and Hone [4] pointed out, the only influence of the localized holes on the spin system is to induce a weaker exchange-interaction across the stripes as compared to those in the magnetic domains. The system is therefore described by a model of coupled Heisenberg spin-ladders, which is in the universality class of the QNLS.

In addition, there are the options that the stripe phase is either bond ordered, as suggested by exact diagonal-

ization studies on the t-J model [5], or site ordered, as suggested by mean field calculations [6]. In spin-only language this would correspond with coupled two-leg and three-leg spin ladders for bond- and site order, respectively. We have performed an extensive analysis of the magnetic properties of these coupled ladder models, finding that the three-leg ladder model can be excluded on basis of the available experimental information. If the stripes are bond-ordered, the physics of coupled two-leg ladders could be responsible for the quantum magnetism of the stripe phase, and a strategy is pointed out to further investigate these matters by experiment.

Before we turn to the microscopy of the coupled-ladder model, let us first discuss the long-wavelength behavior as follows from the QNLS [8]. The QNLS describes the transversal spin-fluctuations of a system with well-formed local magnetic moments. It is characterized by a single coupling constant (g) which measures the strength of the quantum fluctuations. This constant depends on the microscopic details. The behavior of the model as a function of g and temperature is shown in figure 1.

At zero temperature, long-range order is found for g smaller than its critical value, while the Néel state gets disordered at long distances for $g > g_c$. At finite temperatures, four distinct regimes are found, separated by cross-over lines. For small g , the system is in the renormalized classical (RC) region, where the correlation length diverges exponentially as the temperature is lowered towards the long-range ordered phase at $T = 0$. At large coupling, a quantum disordered (QD) region is found, where the system has acquired a spin gap, while the correlation-length becomes independent of temperature. The above two regimes are separated by a quantum critical region. Here, the system behaves as if it is at its zero-temperature critical point on length-scales smaller than the inverse temperature (in appropriate units). The correlation length increases as $1/T$ upon cooling. More generally, the only relevant energy scale in the system is temperature, which gives rise to the so-called ω/T -

scaling (energy scales as temperature). This region terminates at a cut-off temperature, where the correlation length becomes of order of one lattice-spacing. Non-universal effects become important and the continuum QNLS-description is no longer applicable.

Experiments on stripe-ordered systems are consistent with a QNLS-description where g is close to g_c . A first indication is found in measurements of the magnetic ordering temperature. Due to small inter-plane interactions and spin-anisotropies, long-range magnetic order sets in at a finite temperature, when the 2d correlation length reaches a large critical value. In the stripe phase, this occurs at a much lower temperature than in the undoped antiferromagnet [9]. Since the scale on the vertical axis in fig. 1, which is set by the exchange coupling J , is of the same order as for zero doping [10], this suggests a shift of g to a larger value. Further indications come from the work of Kataev *et. al.*, who performed ESR measurements on Eu doped $La_{2-x}Sr_xCuO_4$, using a Gd spin probe [11]. They find an exponential increase of the spin-lattice relaxation time $1/T_1$ upon cooling below the charge-ordering temperature $T_{co} = 70K$, signalling renormalized classical behavior. The correlation length near T_{co} is of order 10 lattice constants. This is larger than the stripe separation, suggesting that a continuum QNLS description is valid, but still much smaller than the correlation length in the undoped system at the same temperature. Again, this suggests that the ordered stripe system is like the undoped antiferromagnet, but with considerably more quantum-fluctuations. For a material which has ordered stripes to high temperatures, we therefore expect to get the behavior indicated by the left grey line in fig. 1.

The question we address is whether our simple spin-only model of the ordered static stripe phase can account for the observed quantum fluctuations in the spin sector. This question, being related to *microscopy* of stripe ordering and thus to non-universal properties of the system, cannot be answered purely on the grounds of continuum field theory. We therefore resort to numerical simulations of the coupled 2- and 3-leg spin ladder system [7]. The highly efficient loop algorithm for the Quantum Monte Carlo method [12] allows us to find the parameter dependences of the correlation length in systems containing up to 1.6×10^4 sites and temperatures as low as $0.03 J$. We extract the cross-over lines separating the different regions of the QNLS diagram from the temperature dependence of the correlation length. Our results are summarized in fig.2. We denote the entrance to the renormalized classical regime by T^* , T' marks the onset of Quantum Disordered behavior, and the "cut-off" T_0 is given by the temperature above which the spin dynamics becomes that of independent single ladders. We also fit our results to the different cross-over scales calculated from an anisotropic QNLS model [7,13], using

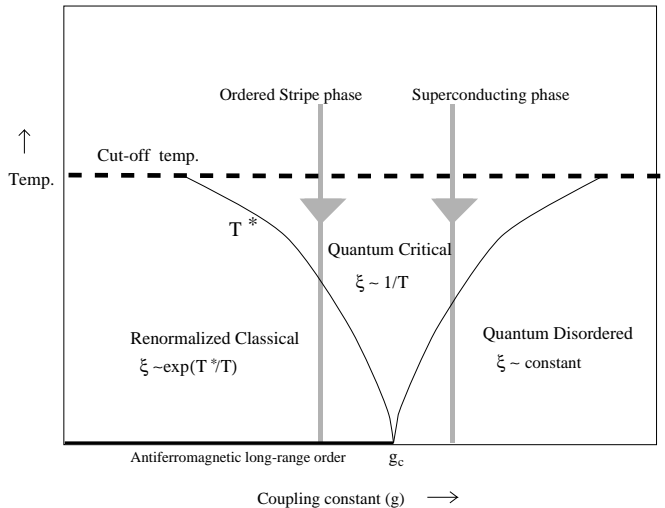


FIG. 1. Cross-over diagram of the QNLS. The grey lines indicate the measurement-trajectories as the system is cooled towards the ordered stripe- and the superconducting phase.

only the bare coupling constant as a fitting parameter.

The isolated 3-leg spin-ladder has a Luttinger liquid ground state, characterized by algebraic decay of the correlations. It is therefore expected that any nonzero inter-ladder coupling αJ (J is the coupling in the ladder) suffices to establish 2+1D antiferromagnetic order [14]. The ground state is then in the renormalized classical regime and the crossover scale T^* acquires a finite value. Our numerical data (Fig.2a) indeed show such a behavior. One interesting observation concerns a non-universal (in the sense of continuum field theory) feature: T^* coincides with the crossover T^0 for any anisotropy α . The classical behavior sets in at the very moment the system discovers that it is (2+1) dimensional. The experimental prediction for such a system would be that no QC behavior could be observed, because the region above RC is always governed by lattice cut-off physics.

This last conclusion turns out to be remarkably different for the 2-leg coupled case. Since the isolated 2-leg ladder possesses an energy gap, the ladder-to-ladder interaction has to overcome this energy scale in order to get a (2+1)D behavior. With our numerical simulations (Fig.2b) we find indeed that the quantum order-disorder transition occurs at a finite and quite large value of anisotropy: $\alpha = 0.30(2)$. In the vicinity of this anisotropy scale the crossovers T^0 and T^* (also T') separate and a *large QC regime opens up*. The results may be quantitatively reproduced with an appropriate [7,13] anisotropic QNLS model (see Fig.2b).

We arrive at the following conclusions: in order to find a strong influence of quantum fluctuations, the ratio of inter- to intra-ladder coupling has to be unrealistically close to zero for the case of site-ordered stripes (3-leg ladders). Hence, the coupled ladder model predicts *bond-ordered* stripes in the cuprates, with α close to its critical

value of 0.3. This gives for the anisotropy in the spin-wave velocities parallel/perpendicular to the stripes

$$\frac{c_{\perp}}{c_{\parallel}} = \sqrt{\frac{2\alpha_c}{1+\alpha_c}} \approx 0.7.$$

The results presented here provide a lowest-order description of the static stripe state. It may be possible to test these results experimentally. The question of site- or bond-ordered stripes can be addressed by NMR. This has already been done for stripes in the Nickelate system $LaNiO_{4+\delta}$, which were found to be site-ordered [15]. Neutron-scattering measurements of the spin-wave velocities along and across the stripes could establish the magnitude of the spatial anisotropy in the spin-spin interactions. This would tell us how much of the quantum fluctuations in the spin-sector can be attributed purely to this anisotropy.

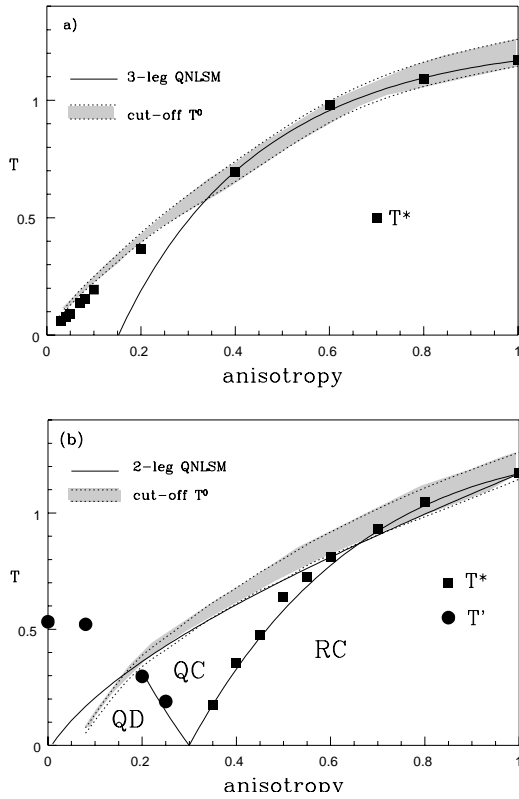


FIG. 2. Temperature crossovers for a) coupled 3-leg ladders b) coupled 2-leg ladders as a function of anisotropy. Solid lines represent the result of fitting an analytic theory.

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